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Effects of Inertia and Thermocapillarity in Non-Isothermal Film Flow

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Abstract

The effect of moving heat source on the flow structure in gravity-driven thin liquid film is studied. The 2-D steady-state conjugated hydrodynamic and thermal problem is solved in long-wave approximation. The flow structures in different regimes are compared: from the regime of flow along vertical substrate with resting heat source to the regime with moving heat source and horizontal liquid layer.

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Keywords: gravity-driven thin liquid film; non-isothermal layer; thermocapillarity; moving local heat source; flow structure

1. Introduction

During local heating of thin liquid films flowing along an inclined plate in experiments there can be observed flow regimes with two- or three-dimensional structure [1]. Due to thermocapillary effect the flow in the heater neighborhood (where a significant liquid temperature gradient arises) undergoes local deceleration that leads to local film thickening. This effect growth in two cases: at flow rate decreasing or at increasing of the heat flux at the heater. When reaching the critical heat flux, a condition of local liquid stop at the free surface takes place. Further heat flux increasing leads to appearance of closed stream lines in the solution of two-dimensional steady-state problem, i.e. a vortex arises inside the area of maximum film thickening. Previously the authors suggested that reaching the local liquid stop leads to limit of stable 2-D flow regimes existence and to development of the transversal instability [2]. As experiments show, as the instability result, new 3-D steady-state regime appears. The film breaks down into periodic rivulets divided by thin liquid layer. In this case dry spots (the film disruptions) can arise on the heater that degrades the heat transfer and brings overheating of the heat releasing devices. On practice this means a danger for microelectronic apparatus destroy when the liquid cooling is used.

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Nomenclature

C	speed of the heat source
c_p	heat capacity of the liquid
$\mathbf{g}$	acceleration due to gravity
h	film thickness
L	length of the heat source
p	pressure
q	heat flux on the heater
Re	Reynolds number
T	temperature
u	longitudinal velocity component of the liquid
$\mathbf{v}$	velocity of the gas
w	transversal velocity component of the liquid
x	coordinate along the substrate
y	coordinate normal to the substrate
Greek symbols	
η	dynamic viscosity of the liquid
θ	angle of substrate inclination relative the horizon
ν	kinematic viscosity of the liquid
ρ	density of the liquid
σ	surface tension
χ	temperature conductivity of the liquid
Subscripts	
H	refers to the heater
0	refers to the initial approximation
∞	refers to the state far before the heater

Thus it is necessary to predict the critical condition when reaching local liquid stop in 2-D regime. The problem under consideration is actual not only for ground applications but for microgravity conditions as well. In the absence of gravity force another mechanism is necessary to provide the relative motion of liquid and the zone of heat release. In some papers a situation was analyzed when one has motionless heater on the substrate and the liquid moves under action of gas flow over the free surface. An alternative scheme is possible: the heat release zone moves relative thin liquid layer on the substrate.

The velocity profiles corresponding to these three cases are shown in Fig. 1. The principal differences in velocity profiles for these three cases are: parabolic (a), linear (b) and uniform (c) profiles. Some combinations of these main types are possible. The sown differences in the velocity profiles lead to significant variations in film deformation and flow structure in non-isothermal conditions. The case (c) is the investigation subject of the present paper. The adjacent hydrodynamic and thermal problem is considered in a common statement that allowed taking into account the gravity effect, the level of which depends not only on the value acceleration due to gravity but on the angle of the substrate inclination as well.

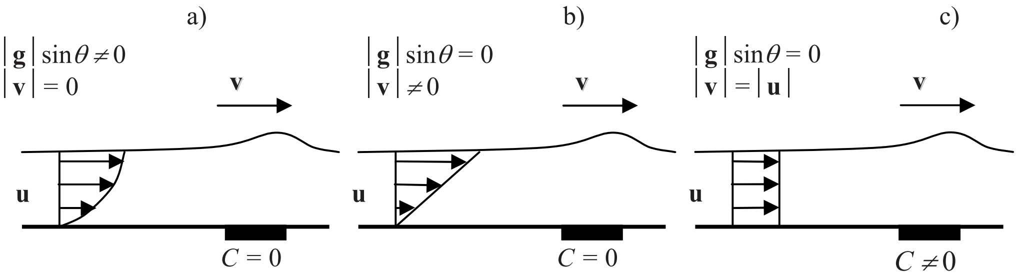

Fig. 1. (a) liquid flow under the gravity action at motionless heater and gas; (b) liquid flow caused by the gas flow over the free surface at motionless heater and zero gravity; (c) liquid flow in frames moving together with the heat release zone in gravity absence (without relative motion between the liquid and gas)

2. Statement of the problem and methods

Theoretical study of non-isothermal film dynamics has resulted in various evolution equations derived for different heat conditions [3]. For the case mentioned above we have some difficulties in formulation of the boundary conditions at the heater. Usually one has either given temperature distribution on the solid surface or given distribution of the heat flux. But in the experiment, as a rule, only integral (average) heat flux over the heater surface is known. This specific feature required development of new approach of numerical solution of the adjacent problem on the structure of 2-D steady-state flow in a locally heated liquid film at lack of information on the heat boundary conditions on the local heater. The matter is in additional iteration cycle fulfilled under assumption on uniform temperature at the heater. This can be justified at high enough temperature conductivity of the heater, i.e. when the characteristic time of the thermal processes in the substrate is much less than one for relaxation processes inside the liquid. Assumption on the temperature uniformity is more realistic than alternative one for heat flux at the heater the value of which decreases quickly along the heater. Nonetheless, unlike the averaged heat flux, mean temperature at the heater isn't specified. Thus the necessity in additional iteration cycle arises. So we use the relaxation method to obtain the numerical solution of a non-linear ordinary differential equation on the liquid film thickness (h):

$$\left(\frac{h^3}{h_\infty^3}-1\right)\sin\theta+\frac{h^3}{h_\infty^3}\left\{\frac{\sigma h_{xxx}}{\rho|\mathbf{g}|}-h_x\cos\theta\right\}+\frac{3h^2\sigma_x}{2\rho|\mathbf{g}|h_\infty^3}=\frac{3\nu C}{|\mathbf{g}|h_\infty^2}\left(\frac{h}{h_\infty}-1\right) \tag{1}$$

where C is speed of the heat source motion, $\mathbf{g}$ is the acceleration due to gravity, θ is the angle of substrate inclination relative the horizon, ν, ρ are the kinematic viscosity and density of the liquid (assumed to be constant), σ is the surface tension, h_∞ is the initial film thickness far from the heater, x is the coordinate along the substrate.

Equation (1) is derived from the Navier-Stokes equations (see [4]) at long-wave approximation and with use of the following boundary conditions: constant flow rate; non-penetration at the substrate $(w=0 \ \text{ at } \ y=0)$; fixed x-component of the velocity on the substrate $(u=-C \ \text{ at } \ y=0)$; kinematic condition on the free surface $(w=uh_x \ \text{ at } \ y=h)$; balance of the forces on the free surface, taking into account for the thermocapillary effect $(\rho\nu u_y=\sigma_x,\ p=p_\infty-\sigma h_{xx} \ \text{ at } \ y=h)$. Interphase transfer between liquid and gas isn't taken into account.

Together with equation (1) on the film thickness one can formulate expressions for pressure (p), longitudinal

(u) and transversal (w) velocity components:

$$p = p^g + (h-y)\rho|\mathbf{g}|\cos\theta - \sigma h_{xx}, \tag{2}$$

$$u = -C + y\sigma_x\eta^{-1} + \left(y^2/2 - hy\right)\left\{h_x\rho|\mathbf{g}|\cos\theta - \rho|\mathbf{g}|\sin\theta - \sigma h_{xxx}\right\}\eta^{-1}, \tag{3}$$

$$w = \frac{y^2 h_x}{2\nu}\{...\} + \frac{y^2(h-y/3)}{2\nu}\{...\}_x - \frac{y^2\sigma_{xx}}{2\rho\nu}, \tag{4}$$

where $\{...\} \equiv \left\{h_x|\mathbf{g}|\cos\theta - |\mathbf{g}|\sin\theta - \rho^{-1}\left(\sigma h_{xx}\right)_x\right\}$.

The thermal part of the problem includes the heat transfer equation

$$\chi\left(T_{xx} + T_{yy}\right) = uT_x + wT_y, \tag{5}$$

with following boundary conditions: the temperature (T) of ram liquid equals T_∞, the heat flux on the wall outside the heater is zero, the averaged heat flux on the heater equals q, density, viscosity and temperature conductivity (χ) of the liquid are considered as non-depending on the temperature, the interface heat transfer and momentum transfer at the free surface are neglected. As the initial approximation for the solution of adjacent problem a flat liquid film is taken with temperature T_∞ at the temperature on the heater $T_H = T_{H0}$.

At each of the "external" cycle iteration first the entire iteration cycle is fulfilled. The velocity field is calculated using the known dependence $h(x)$ and equations (3), (4). After that the explicit finite-difference scheme is applied to find temperature field according to equation (5). Temperature distribution at the free surface determines the surface tension gradient in (1). The solution of this equation with constant film thickness far from the heater can be calculated using finite-difference scheme and allows start new iteration. The entire iterative process is repeated until the square deviation of the solution on the iteration step would become smaller then a preset small value (10^{-6}). The obtained data allow one to determine the local heat flux at the heater. If the mean heat flux is less (or higher) than q, then new entire iteration cycle is carried out with mean temperature at the heater higher (or less) than previous value. However the accuracy of this method isn't high. The other, more accurate method for correction in T_H is based on comparison of pre-assigned integral heat flux and full change of heat power of the flow. As it is assumed, all the heat from the heater is spent on the heating of the liquid only. The temperature of the liquid is homogeneous far from the heater, so one can find the amount of heat which has to be released per unite time in this regime with constant flow rate. If this quantity is less (or higher) then the assigned value q, then it needs to repeat the entire iteration cycle with a higher (less) mean temperature at the heater T_H. Thus the external iteration cycle is realized and its completion corresponds to the establishment of the balance of heat fluxes.

3. Results and discussion

Although the experiments on capillary film deformation with moving local heat source haven't been carried out, nevertheless physical parameters used in calculations correspond to the known experiments with gravity-driven film and immobile heater (25% solution of ethyl alcohol in water): $T_\infty = 303$ K; dynamic viscosity $\eta = 1.7\cdot10^{-3}$ kg/m s; heat capacity $c_p = 4.3\cdot10^3$ J/kg K; $\chi = 10^{-7}$ m^2/s; $d\sigma/dT = -1.1\cdot10^{-4}$ kg/s^2 K; $\sigma_\infty = 35.4\cdot10^3$ kg/s^2; Reynolds number Re $= 2$ (flow rate equals to η Re); $h_\infty = 1.25\cdot10^{-4}$ m; $|\mathbf{g}| = 9.8$ m/s^2; $q = 2.72$ W/cm^2; the length of the heater $L = 6.5\cdot10^{-3}$ m.

Thermocapillary deformation of the film (Fig. 2), fields of the velocity and temperature under different C and θ, but at the same values of Re, h_∞, q, L were calculated. The local arrest of the liquid is achieved at $\theta \approx 3.8°$. If $\theta < 3.8°$, then a vortex presents in the flow structure. The characteristic pattern of stream lines is represented in Fig. 3 for different level of gravity. At the other equal conditions the flow structure depends strongly on the type of velocity profile in the flat film. Substitution of the parabolic velocity profile with uniform one is followed by the dramatic film deformation increase, and the flow velocity at the free surface is reduced near high surface tension gradient, the longitudinal component of the velocity even changes the sign, which corresponds to the vortex (probably it means instability of 2-D steady-state regime).

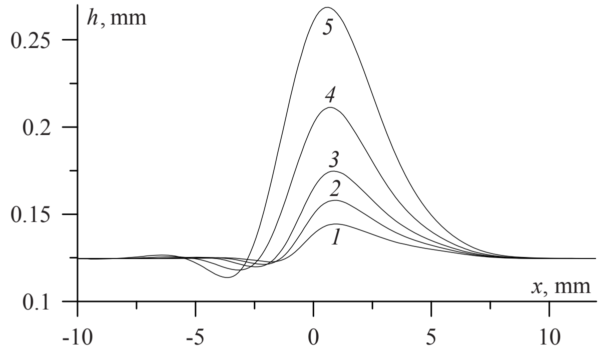

Fig. 2. Free surface at: (1) $C = 0$, $\theta = 90°$; (2) $C = -8.36$ mm/s, $\theta = 45°$; (3) $C = -21.15$ mm/s, $\theta = 15°$; (4) $C = -26.54$ mm/s, $\theta = 4°$; (5) $C = -28.54$ mm/s, $\theta = 0°$($x = 0$ corresponds to the upper edge of the heater)

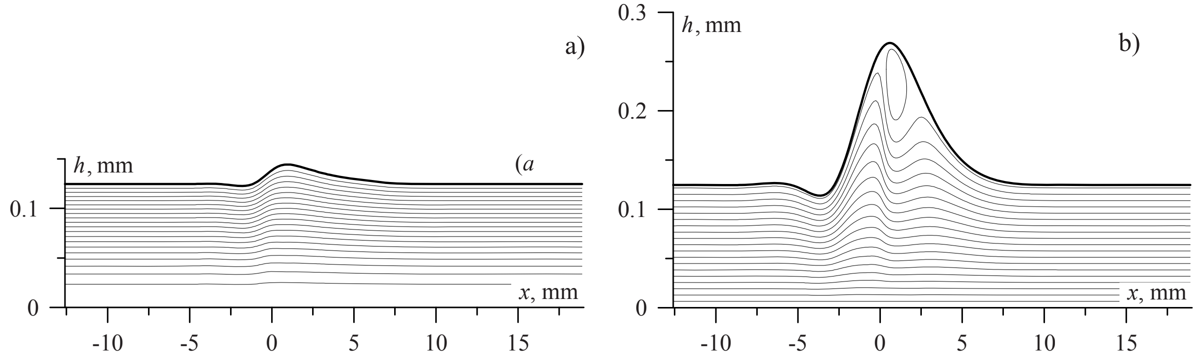

Fig. 3. streamlines at $q = 2.7$ W/cm^2 and Re = 2: (a) $C = 0$, $|\mathbf{g}| = 9.8$ m/s^2; (b) $C = -0.285$ m/s, $|\mathbf{g}| = 0$

Changing of θ models different levels of gravity (with the same other conditions). The obtained analytical and numerical data on the vortex appearance in film flow with local heating were analyzed and generalized. The results are represented in the Fig. 4 (θ and C are connected by the condition of constant flow rate).

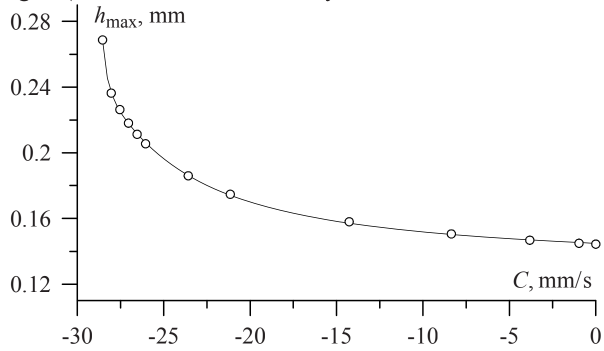

Fig. 4. dependence of h_{max} on C: (points) results of calculations; (line) polynomial interpolation

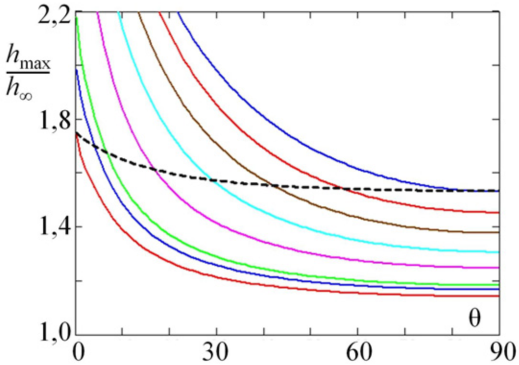

Fig. 5. dependence of maximum relative film thickening on the inclination angle at Re = 2 and q = 2.3, 2.7, 3, 4, 5, 6, 7, 8 W/cm^2; dashed line corresponds to the critical regimes and separates the flows with vortex (above) and without vortex (below)

The calculations with different q allow us to analyze the dependence of thermocapillary thickening of the film on the inclination angle (or on the speed of the heat source). Fig. 5 represents the maximal relative film thickening (Re = 2) and the critical regimes, which separate the flow with vortex. The results show that the inclination angle corresponding to critical regime increases with heat flux increase. If $q < 2.3$ W/cm^2, then critical regime absent even at $\theta = 0^\circ$. On the other hand, there exists the level of the heat flux (~ 8 W/cm^2), when 2-D steady-state flow regime without vortex is impossible. Such a value of heat flux can be compared with experimental data [5] on critical heat flux at different flow rates, and $\theta = 90°$.

Analysis of the Fig. 5 reveals that in the critical regimes $\sin(\theta) \sim q$. It is remarkable that the experiments [5] with gravity-driven film and $C = 0$ also demonstrate the linear dependency $q(\sin(\theta))$ for the critical regimes, when the instability of 2-D flow appears.

Acknowledgements

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References

[1] Kabov OA, Diatlov AV, Tereshchenko AG. Heat transfer from a small heater to the falling film of the mixture of ethyl alchohol in water. *Thermophys. Aeromech.* 1996;**3**(1):31 – 44.
[2] Sharypov OV, Medvedko KA. On the stability of a 2-D film flow regime with a non-uniform temperature of the free surface. *Russ. J. Eng. Thermophys*. 2000;**10**(4):315 – 36.
[3] Joo SW, Davis SH, Bankoff SG. Long-wave instabilities of heated falling films: two-dimensional theory of uniform layers. *J. Fluid Mech*. 1991;**230**:117 – 46.
[4] Sharypov OV, Kuibin PA. Heat-wave induced vortex in a thin liquid layer. *Int. Rev. Chem. Eng.* 2009;**1**:158 – 63.
[5] Kabov OA. *Influence of capillary effects on film condensation and heat transfer in liquid films.* Doctor of Sci. Theses, Novosibirsk: Kutateladze Institute of Thermophysics SB RAS; 1999.